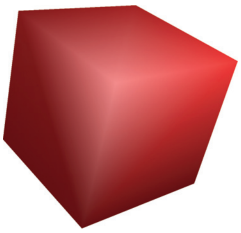
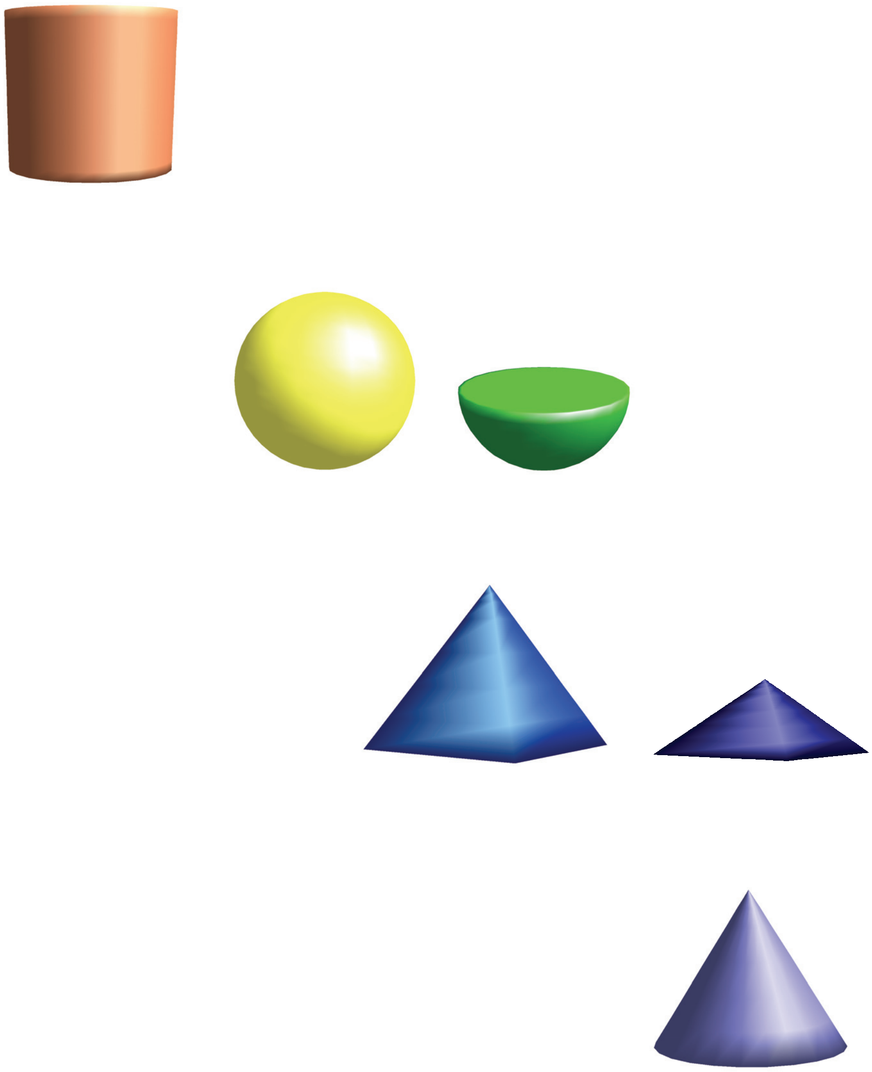


CUBE GEOMETRY



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PREFACE

While I was working on The Cylinder Book my attention was drawn to one of the five platonic solids: the regular hexahedron.
The first calculations lead to the idea of 'maximum cylinder' which is the maximum volume of a cylinder inscribed in a regular hexahedron.
After that I decided to take a look at some other solids which fit into a cube.

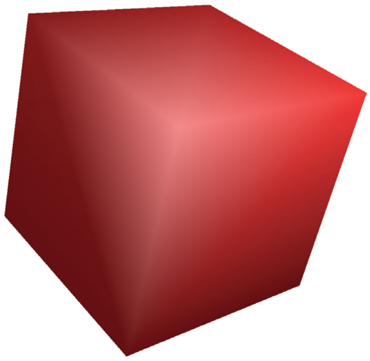
C R Thonus 2007

CONTENTS

RATIOS

Let's start with the regular hexahedron:

A regular polyhedron for which all faces are squares. It is one of the five platonic solids.



Since we take the cube as base for our work we can give its volume a simple value: 1m³

This is not strictly necessary, you may choose any value you like for the cube volume, the ratio of the volumes will always be exactly the same.

Now let's take a look at the largest right circular cylinder which fits exactly into our cube.

This cylinder will have following dimensions:

Length: 1 m   Radius circular ends: 0,5 m

The volume of this cylinder is: 3,14159265358979*0,5*0,5*1

V cyl. =   0,785398163397448 m³

Another example: A cube with volume 268,348 m³

The largest right circular cylinder which fits exactly into this cube:

Edge cube= 6,450095145 m

This cylinder will have following dimensions:

Length: 6,450095145 m   Radius circular ends: 3,225047573 m

The volume of this cylinder is: 3,14159265358979*3,225047573 Λ 2*6,450095145

V cyl. =   210,760026404756 m³

Ratio volume cylinder / volume cube= 210,760026404756/268,348=

0,785398163359636

The difference is due to rounding errors.

Next one on our list: the sphere.

The largest sphere fitting into our cube of 1 m³ will have following dimensions:

Radius: 0,5 m

The volume of this sphere is: 4/3 π*r*r*r

V sphere = 0,523598775598298 m³

Next one: right pyramid with a square base and height equal to the side of its base.

The largest right pyramid fitting into our cube of 1 m³ will have following dimensions:

Height: 1 m Side base: 1 m

The volume of this pyramid is: 1/3*1*1*1

V right pyramid = 0,333333333333333 m³

The last one is a right circular cone with height equal to the diameter of its circular base.

The largest right circular cone fitting into our cube of 1 m³ will have following dimensions:

Height: 1 m Diameter base: 1 m

The volume of this cone is: 1/3*3,14159265358979*0,5*0,5*1

V right circular cone = 0,261799387799149 m³

SUMMARY

The basis of our system is a cube of 1 m^3

The volume of the right circular cylinder which fits exactly into this cube=

0,785398163397448 m^3

The volume of the sphere which fits exactly into this cube=

0,523598775598298 m^3

The volume of the right pyramid with square base which fits exactly into this cube=

0,333333333333333 m^3

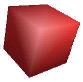
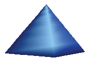
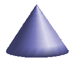
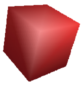
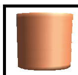
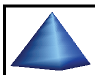
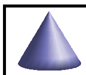
The volume of the right circular cone which fits exactly into this cube=

0,261799387799149 m^3

The ratio of the volume of any cube and the volume of one of these four objects which fits exactly into this cube has always the same value.

In other words: for any given cube volume we can deduce the corresponding inscribed object volume by simply multiplying with the appropriate factor.

TABLE OF RATIOS

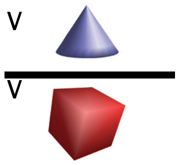
	Volume					
↑		1		0,5235987		0,2617993
→→→→ *			0,7853981		0,3333333	
↑		1,2732395		0,6666666		0,3333333
→→→→ *			1		0,4244131	
↑		1,9098593		1		0,5
→→→→ *			1,5		0,6366197	
↑		3		1,5707963		0,7853981
→→→→ *			2,3561944		1	
↑		3,8197186		2		1
→→→→ *			3		1,2732395	

IN WORDS

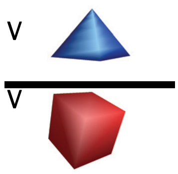
Remember: what follows applies to objects INSCRIBED in a regular hexahedron!

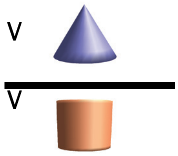
1	VOLUME CUBE	* 0,785398163397448 =	VOLUME CYLINDER
2	VOLUME CUBE	* 0,523598775598298 =	VOLUME SPHERE
3	VOLUME CUBE	* 0,333333333333333 =	VOLUME PYRAMID
4	VOLUME CUBE	* 0,261799387799149 =	VOLUME CONE
5	VOLUME CYLINDER	* 1,27323954473516 =	VOLUME CUBE
6	VOLUME CYLINDER	* 0,666666666666666 =	VOLUME SPHERE
7	VOLUME CYLINDER	* 0,424413181578388 =	VOLUME PYRAMID
8	VOLUME CYLINDER	* 0,333333333333333 =	VOLUME CONE
9	VOLUME SPHERE	* 1,90985931710275 =	VOLUME CUBE
10	VOLUME SPHERE	* 1,5 =	VOLUME CYLINDER
11	VOLUME SPHERE	* 0,636619772367582 =	VOLUME PYRAMID
12	VOLUME SPHERE	* 0,5 =	VOLUME CONE
13	VOLUME PYRAMID	* 3 =	VOLUME CUBE
14	VOLUME PYRAMID	* 2,35619449019234 =	VOLUME CYLINDER
15	VOLUME PYRAMID	* 1,57079632679489 =	VOLUME SPHERE
16	VOLUME PYRAMID	* 0,785398163397448 =	VOLUME CONE
17	VOLUME CONE	* 3,81971863420549 =	VOLUME CUBE
18	VOLUME CONE	* 3 =	VOLUME CYLINDER
19	VOLUME CONE	* 2 =	VOLUME SPHERE
20	VOLUME CONE	* 1,27323954473516 =	VOLUME PYRAMID

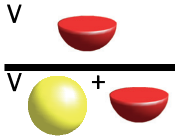
IN SYMBOLS

1) 

== [0,261799387799149]

2) 

== 

== 

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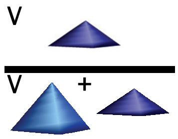
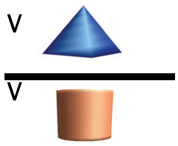
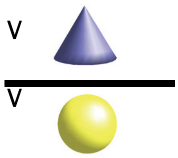


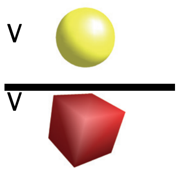
== [0,333333333333333]

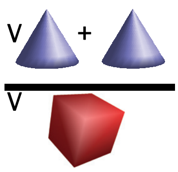
3) 

== [0,424413181578388]

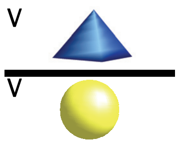
4) 

== [0,5]

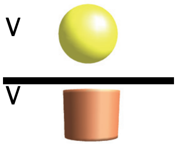
5) 

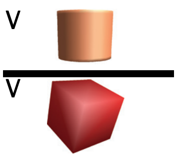
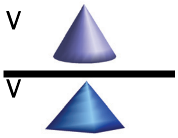
== 

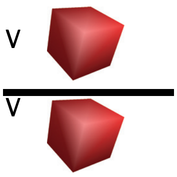
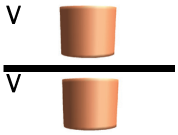
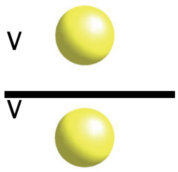
== [0,523598775598298]

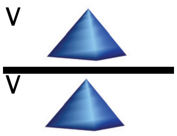
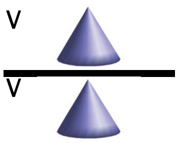
6) 

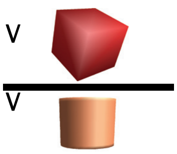
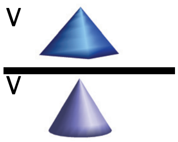
== [0,636619772367582] == 2/pi

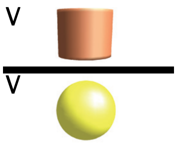
7)  == [0,666666666666666]

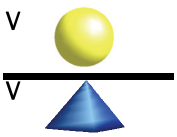
8)  ==  == [0,785398163397448]

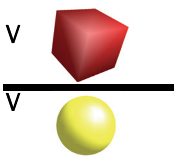
9)  ==  ==  ==

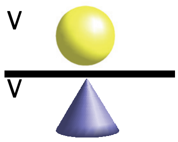
 ==  == [1]

10)  ==  == [1,27323954473516]

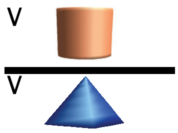
11)  == [1,5]

12)  == [1,57079632679489] == pi/2

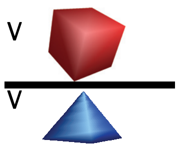
13)  == [1,90985931710275]

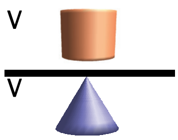
14) 

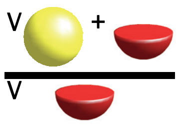
== [2]

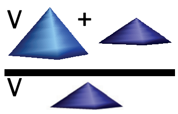
15) 

== [2,35619449019234]

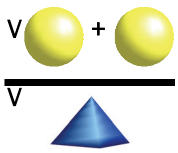
16) 

== 

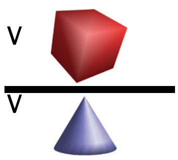
== 

== 

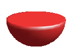
== [3]

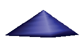
17) 

== [3,14159265358979]

18) 

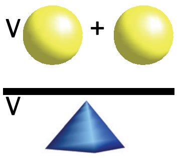
== [3,81971863420549]

 = Half sphere

 = Half right pyramid with square base (½ height cube, half the volume of a right square pyramid with square base and height cube)

V = Volume

3,14159265358979323846264338327 9...



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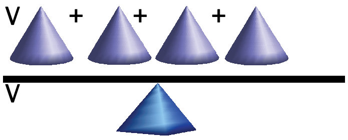
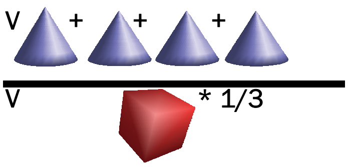


== 3,1415926535897932384626...



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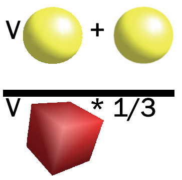
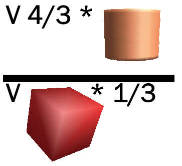
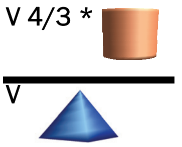


== 3,14159265358979323846264338327 9...



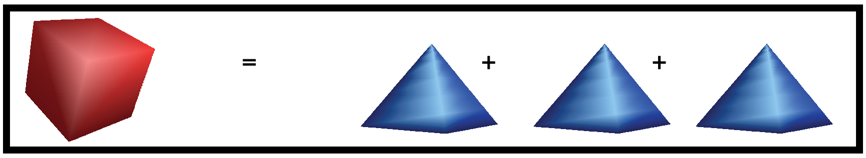
== 3,14159265358979323846264338327 9...



== 3,14159265358979323846264338327 9...

THREE BASIC EQUATIONS OF CUBE GEOMETRY

1)



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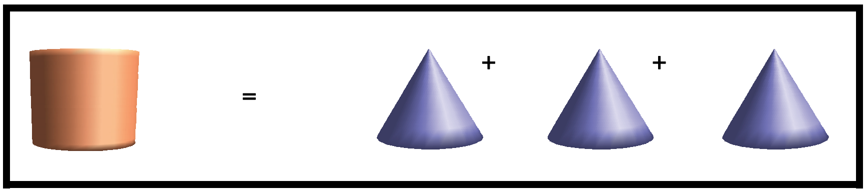
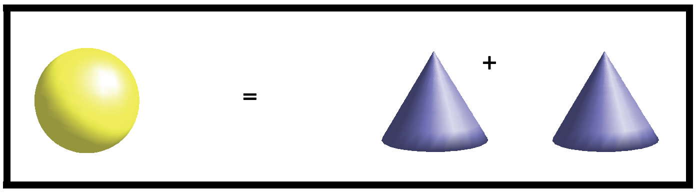


3)



MORE ABOUT PYRAMIDS

In our cube system we can easily make a formula for all right pyramids with a square base :

$$\text{Volume pyramid (A)} = \frac{\text{Height pyramid (A)}}{\text{Length side base (B)}} * \text{Volume pyramid base (B)}$$

Example: Right pyramid with square base

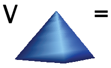
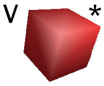
Height : 0,13 m

Length side base : 1,26 m = also length of edge base cube

$$\text{Volume} = \frac{0{,}13 \qquad * 1{,}26^3}{1{,}26 \; * \qquad 3} \quad * 1$$

$$= \frac{0{,}13 \qquad * 1{,}26^2}{3} \quad * 2$$

$$= 0{,}068796 \text{ m}^3$$

* 1 Remember: V  = V  * 1/3

$$* 2 \quad \frac{0{,}13 \qquad * 1{,}26^2}{3} = \frac{\text{height * area base of cube}}{3}$$

$$\text{Volume right pyramid square base} = \frac{\text{height * area base of cube}}{3}$$

$$\text{Volume right pyramid square base} = \frac{\text{Height * side base}^2}{3}$$

MORE ABOUT CONES

In our cube system we can also easily make a formula for all right cones with a circular base :

$$\text{Volume cone (A)} = \frac{\text{Height cone (A)}}{\text{Diameter base (B)}} * \text{Volume cone Ø base (B)}$$

Example: Right cone with circular base

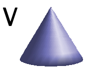
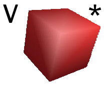
Height : 1,83 m

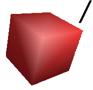
Ø base : 0,76 m = also length of edge base cube

$$\text{Volume} = \frac{1{,}83 \quad * 0{,}76^3}{0{,}76 \; * \; 3{,}8197186} \quad * 1$$

$$= \frac{1{,}83 \quad * 0{,}76^2}{3{,}8197186} \quad * 2$$

$$= 0{,}2767240 \text{ m}^3$$

* 1 Remember: V  = V  * 0,261799387799149

= V  / 3,81971863420549

* 2 $\frac{1{,}83 \quad * 0{,}76^2}{3{,}8197186} = \frac{\text{height * area base of cube}}{3{,}8197186}$

$$\text{Volume right circular cone} = \frac{\text{height * area base of cube}}{3{,}81971863420549}$$

$$\text{Volume right circular cone} = \frac{\text{Height * Ø base}^2}{3{,}81971863420549}$$

TESTING

For our test we take a randomly chosen cube with edge 2,66 m.

V CUBE = 18,821096 m^3

1) V maximum cylinder = 18,821096 *0,785398163397448

= 14,782054 m^3

2) V maximum cylinder = 3,1415926 * $1{,}33^2$ * 2,66

= 14,782054 m^3

1) V maximum sphere = 18,821096 * 0,523598775598298

= 9,8547028 m^3

2) V maximum sphere = 3,1415926 * $1{,}33^3$ * 4/3

= 9,8547027 m^3

1) V maximum pyramid = 18,821096 *0,333333333333333

= 6,2736987 m^3

2) V maximum pyramid = 1/3 * $2{,}66^2$ * 2,66

= 6,2736987 m^3

1) V maximum cone = 18,821096 *0,261799387799149

= 4,9273514 m^3

2) V maximum cone = 1/3 * 3,1415926 * $1{,}33^2$ * 2,66

= 4,9273513 m^3

1) Calculation with the cube method.

2) Calculation with the standard method.

CYLINDERS WITH SEMI-SPHERICAL ENDS

EXAMPLE :

Length of cylinder: 4,15 m

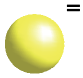
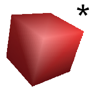
Radius sphere: 0,6 m

Overall length: 5,35 m

Total volume = V cube * 0,523598775598298 + V cylinder

= $1{,}2^3$ * 0,523598775598298 + (4,15 * $1{,}2^2$ * 0,785398163397448)

= 5,5983181 m^3

Remember : V  = V  * 0,523598775598298

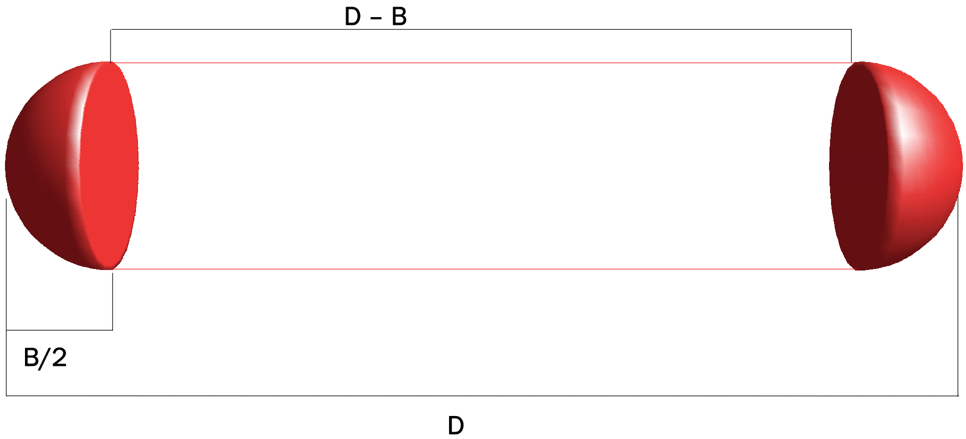
V right circular cylinder with semi-spherical ends :

V = $(R*2)^3$ * 0,523598+[L*$(R*2)^2$ * 0,785398]

R = radius semi-spherical end L = length cylinder

RIGHT SQUARE PRISM WITH FOUR SEMI-SPHERICAL LATERAL FACES

Let's start with the basic form of our *square sphere plane:* a cylinder with semi-spherical ends formed by the displacement of a sphere with diameter B.



Length of cylinder = displacement sphere = D – B

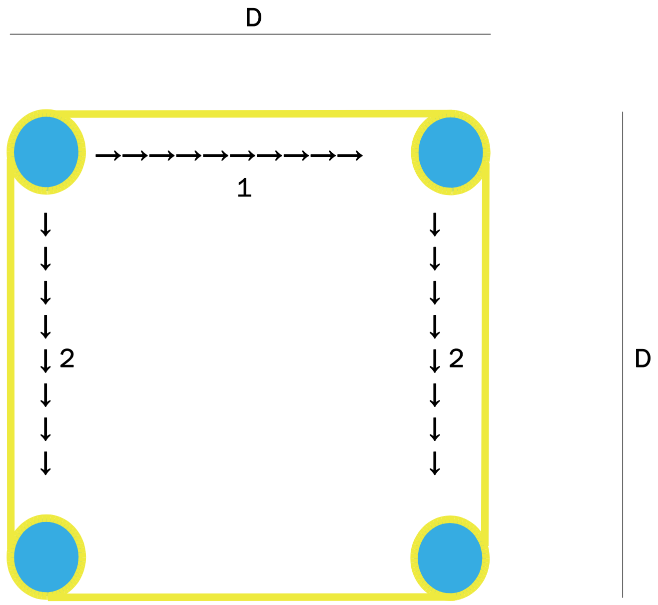
Overall length = D

Diameter sphere = B

We now move our object perpendicular to length D over a distance D – B and in the same plane.

What we get:

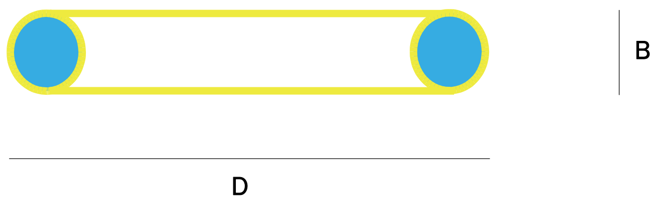
Top view:



1 = first displacement of sphere over distance D – B

2 = second displacement of cylinder with semi-spherical ends in a plane over distance D – B

Side view:



Calculation of the volume of our object with the cube method:

$$V = [(D - B)^2 * B] + B^3 * 0{,}523598 + 2[(D - B) * B^2 * 0{,}785398]$$

$(D - B)^2 * B$ = Volume of rectangular parallelepiped left after truncating 1 sphere and 2 cylinders.

$B^3 * 0{,}523598$ = Volume of sphere left after truncating $(D - B)^2 * B$ and 2 cylinders.

$2[(D - B) * B^2 * 0{,}785398]$ = Volume of the 2 cylinders left after truncating $(D - B)^2 * B$ and 1 sphere.

EXAMPLE : B = 0,1 m D = 1,2 m

$V = [(1{,}2-0{,}1)^2 * 0{,}1]+0{,}1^3 * 0{,}523598+2[(1{,}2-0{,}1)*0{,}1^2 * 0{,}785398]$

$V = 0{,}138802354\ m^3$

If our object was a right square prism *WITHOUT* four semi-spherical lateral faces and with the same dimensions, the volume would be:

$V = 1{,}2*1{,}2*0{,}1$

$V = 0{,}144\ m^3$

RECTANGULAR PARALLELEPIPED WITH FOUR SEMI-SPHERICAL LATERAL FACES

Let's start with the basic form of our *rectangular sphere plane:* a cylinder with semi-spherical ends formed by the displacement of a sphere with diameter B.

Length of cylinder = displacement sphere = D – B

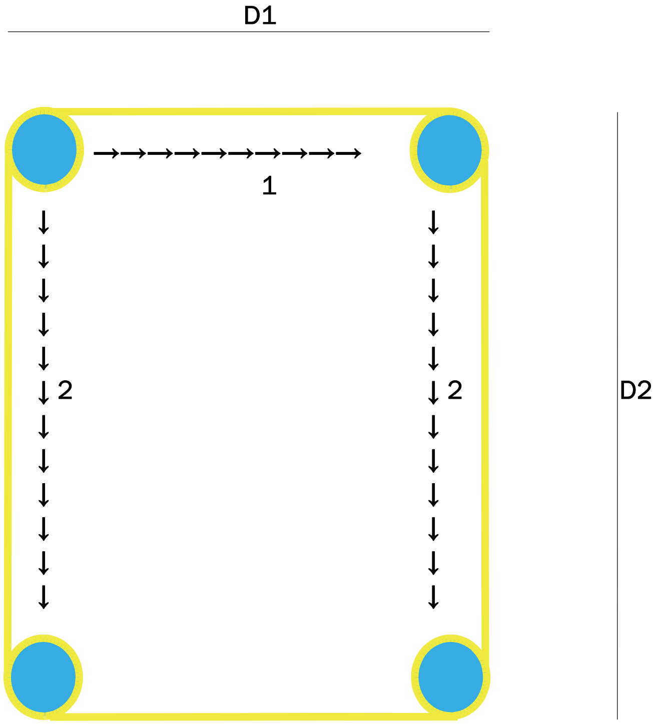
Overall length = D

Diameter sphere = B

We now move our object perpendicular to length D over a distance D – B + x (x > 0) and in the same plane.

What we get:

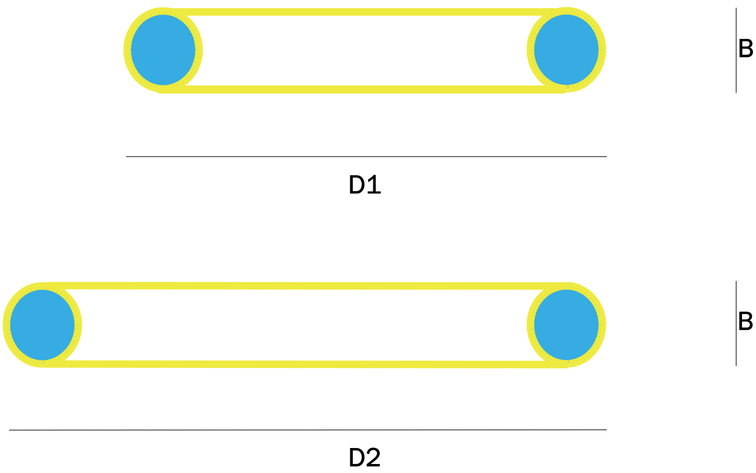
Top view:



1 = first displacement of sphere over distance D – B

2 = second displacement of cylinder with semi-spherical ends in a plane over distance D – B + x

Side view:



Calculation of the volume of our object with the cube method:

$$V = (D1 - B * D2 - B * B) + [B^3 * 0{,}523598 + (D1 - B) * B^2 * 0{,}785398] + [(D2 - B) * B^2 * 0{,}785398]$$

(D1 – B * D2 – B * B)= Volume of rectangular parallelepiped left after truncating
1 (D1 – B) cylinder, 2 semi-spherical ends and 1 (D2 – B) cylinder.

$[B^3 * 0{,}523598 + (D1 - B) * B^2 * 0{,}785398]$=
Volume of 1 (D1 - B) cylinder and 2 semi-spherical ends left after truncating
(D1 – B * D2 – B * B) and 1 (D2 - B) cylinder.

$[(D2 - B) * B^2 * 0{,}785398]$= Volume of 1 (D2 – B) cylinder left after truncating
(D1 – B * D2 – B * B) and $[B^3 * 0{,}523598 + (D1 - B) * B^2 * 0{,}785398]$.

EXAMPLE : B = 0,1 m D1 = 1,2 m D2 = 1,8 m

$$V = (1{,}2-0{,}1*1{,}8-0{,}1*0{,}1)+[0{,}1^3 * 0{,}523598+(1{,}2-0{,}1)*0{,}1^2 * 0{,}785398]+[(1{,}8-0{,}1)*0{,}1^2 * 0{,}785398]$$

$V = 0{,}209514542\ m^3$

If our object was a rectangular parallelepiped *WITHOUT* four semi-spherical lateral faces and with the same dimensions, the volume would be:

$V = 1{,}2*1{,}8*0{,}1$

$V = 0{,}216\ m^3$

REGULAR HEXAHEDRON WITH ¼ SPHERICAL EDGES

(sphere cube)

Let's take a right square prism with four semi-spherical lateral faces:

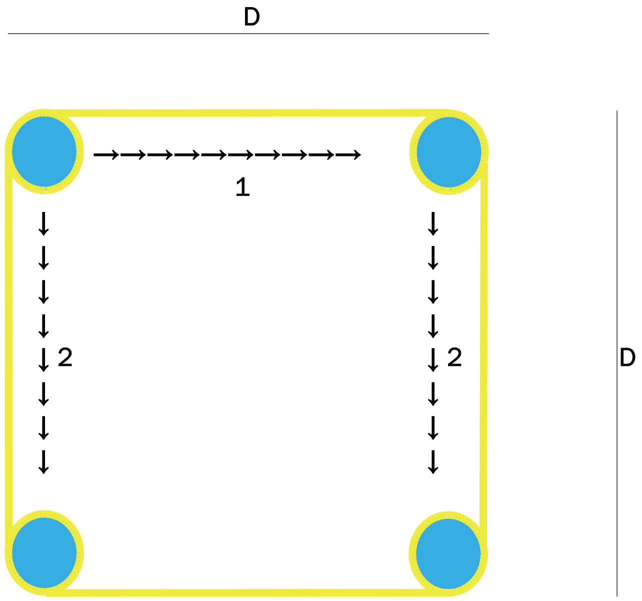
1 = first displacement of sphere over distance D – B

2 = second displacement of cylinder with semi-spherical ends in a plane over distance D – B

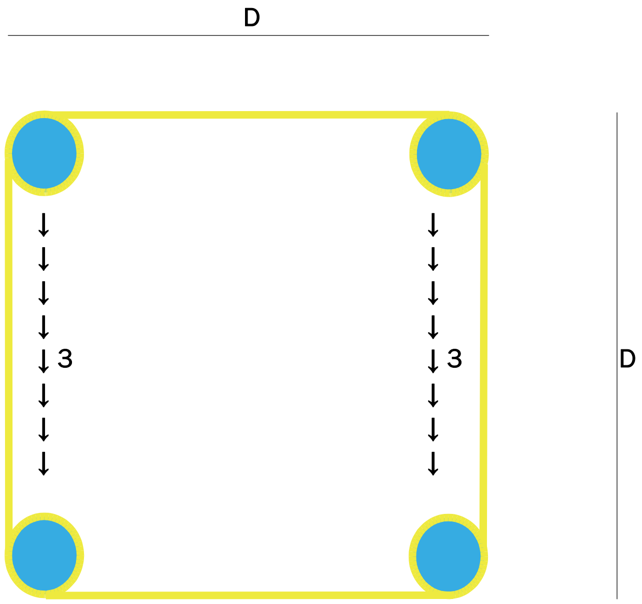
3 = third displacement of right square prism with four semi-spherical lateral faces over distance D - B perpendicular to the plane of the object

What we get:

Top view:



Side view:



Calculation of the volume of our object with the cube method:

$$V = B^3 * 0,523598 + 3[(D - B)*B^2 * 0,785398] + (D - B)^2 * D + 4[(D - B)^2 * B/2]$$

$B^3 * 0,523598$ = Volume of 1 sphere left after truncating 3 (D – B) cylinders, 1 $(D - B)^2 * D$ rectangular parallelepiped and $4[(D - B)^2 * B/2]$ rectangular parallelepiped.

$3[(D - B)*B^2 * 0,785398]$ = Volume of 3(D – B) cylinders left after truncating 1 sphere and 1+4 rectangular parallelepiped.

$(D - B)^2 * D$ = Volume of the middle rectangular parallelepiped left after truncating 1 sphere, 3(D – B) cylinders and the 4 outermost rectangular parallelepiped.

$4[(D - B)^2 * B/2]$ = Volume of the 4 outermost rectangular parallelepiped left after truncating 1 sphere, 3(D – B) cylinders and the middle rectangular parallelepiped.

EXAMPLE : B = 0,1 m D = 1,2 m

$$V = 0,1^3 * 0,523598 + 3[(1,2-0,1)*0,1^2 * 0,785398] + (1,2-0,1)^2 * 1,2 + 4[(1,2-0,1)^2 * 0,1/2]$$

$V = 1,720441732\ m^3$

If our object was a regular hexahedron *WITHOUT* ¼ spherical edges
and with the same dimensions, the volume would be:

$V = 1,2^3$

$V = 1,728\ m^3$

RECTANGULAR PARALLELEPIPED WITH ¼ SPHERICAL EDGES

(sphere rectangular parallelepiped)

Let's take a rectangular parallelepiped with four semi-spherical lateral faces:

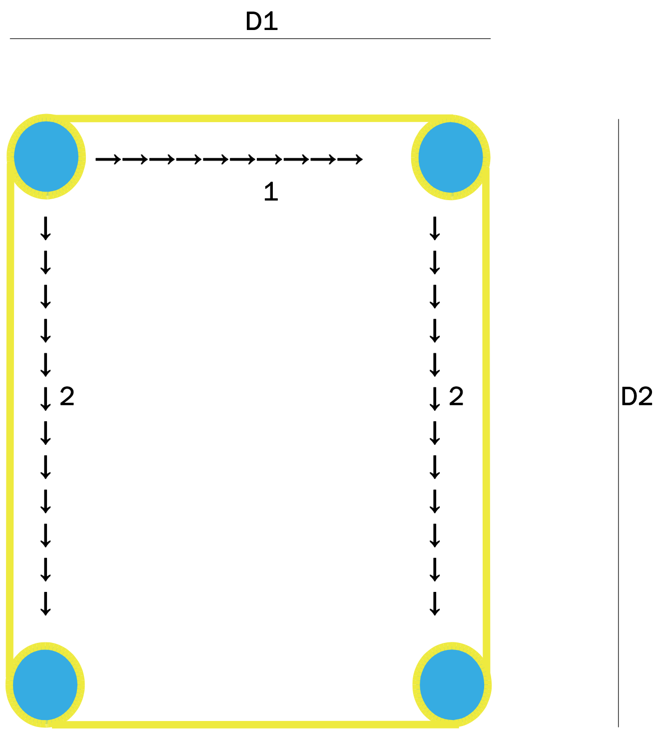
1 = first displacement of sphere over distance D – B

2 = second displacement of cylinder with semi-spherical ends in a plane over distance D – B + x

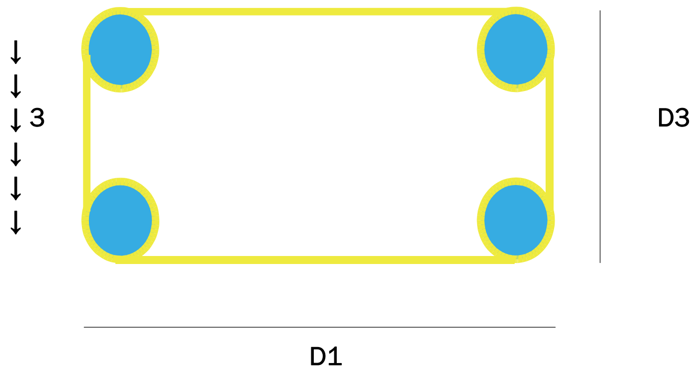
3 = third displacement of rectangular parallelepiped with four semi-spherical lateral faces over distance y perpendicular to the plane of the object

What we get:

Top view:



Side view:



Calculation of the volume of our object with the cube method:

V= B³ * 0,523598+[(D1-B)*B² * 0,785398]+[(D2-B)*B² * 0,785398]+[(D3-B)*B² * 0,785398]

+[D1-B*D2-B*D3]+2(D2-B*D3-B*B/2)+2(D1-B*D3-B*B/2)

B³ * 0,523598 = Volume of 1 sphere left after truncating 3 cylinders (D1-B, D2-B, D3-B), 1 middle rectangular parallelepiped and 4 outermost rectangular parallelepiped.

[(D1-B)*B² * 0,785398]+[(D2-B)*B² * 0,785398]+[(D3-B)*B² * 0,785398] = Volume of 3 cylinders left after truncating 1 sphere, 1 middle rectangular parallelepiped and 4 outermost rectangular parallelepiped.

[D1-B*D2-B*D3]+2(D2-B*D3-B*B/2)+2(D1-B*D3-B*B/2) = Volume of 1 middle rectangular parallelepiped and 4 outermost rectangular parallelepiped left after truncating 1 sphere and 3 cylinders (D1-B, D2-B, D3-B).

EXAMPLE : B = 0,1 m D1 = 1,2 m D2 = 1,8 m D3 = 1,4 m

$V= 0,1^3 * 0,523598+[(1,2-0,1)*0,1^2 * 0,785398]+[(1,8-0,1)*0,1^2 * 0,785398]+$
$[(1,4-0,1)*0,1^2 * 0,785398]+[1,2-0,1*1,8-0,1*1,4]+2(1,8-0,1*1,4-0,1*0,1/2)+$
$2(1,2-0,1*1,4-0,1*0,1/2)$

$V= 3,014724916\ m^3$

If our object was a rectangular parallelepiped *WITHOUT* ¼ spherical edges and with the same dimensions, the volume would be:

$V= 1,2*1,8*1,4$

$V= 3,024\ m^3$